

Entanglement of pair cat states and teleportation

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Abstract

The entanglement of pair cat states in the phase damping channel is studied by employing the relative entropy of entanglement. It is shown that the pair cat states can always be distillable in the phase damping channel. Furthermore, we analyze the fidelity of teleportation for the pair cat states by using joint measurements of the photon-number sum and phase difference.

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I. INTRODUCTION

Quantum entanglement plays an important role in various fields of quantum information, such as quantum computation [1], quantum cryptography [2], quantum teleportation [3,4], dense coding [5] and quantum communication [6], etc. It has been recognized [7,8] that quantum teleportation can be viewed as an achievable experimental technique to quantitatively investigate quantum entanglement. There exists a class of states called maximally correlated states, which have an interesting property, i.e., the PPT distillable entanglement is exactly the same as the relative entropy of entanglement [9]. Both two-mode squeezed vacuum states and pair cat states [10] belong to this class. In continuous variable teleportation the entanglement resource is usually the two-mode squeezed state, or the Einstein-Podolsky-Rosen (EPR) states. Continuous variable quantum teleportation of arbitrary coherent states has been realized experimentally by employing a two-mode squeezed vacuum state as an entanglement resource [7]. Theoretical proposals of teleportation scheme based on the other continuous variable entangled states have already been discussed [11]. Update now, little attention has been paid to the entanglement properties of the pair cat state and its possible application of quantum information. Gou et al. have proposed a scheme for generating the pair cat state of motion in a two-dimensional ion trap [12]. In their scheme, the trapped ion is excited bichromatically by five laser beams along different directions in the X-Y plane of the ion trap. Four of these have the same frequency and can be derived from the same source, reducing the demands on the experimentalist. It is shown that if the initial vibrational state is given by a two-mode Fock state, pair cat states are realized when the system reaches its steady state. Their work motivates us to investigate the entanglement properties of the pair cat state and its possible application in quantum information processes, such as quantum teleportation. The motivation is two-fold: (1) the storage of continuous variable entangled states in two-dimensional motional states of trapped ions is feasible in current experimental techniques. (2) the mapping of steady state entanglement to optical beams is also realizable [13].

On the other hand, quantum entanglement is a fragile nature, which can be destroyed by the interaction between the real quantum system and its environment. This effect, called decoherence, is the most serious problem for all entanglement manipulations in quantum information processing. There have several proposals for entanglement distillation and purification in continuous variable systems [14]. In this paper, we firstly investigate the relative entropy of entanglement of pair cat states in the phase damping channel, and show that the pair cat states can always be distillable in the phase damping channel. Then, we explore possible application of pair cat states in quantum information processing, such as quantum teleportation. The fidelity of teleportation protocol in which the mixed pair cat state is used as a entangled resource, is analyzed.

This paper is organized as follows. In section II, based on the exact solution of the master equation describing phase damping, we give the numerical calculations of relative entropy of entanglement for pair cat states in the phase damping channel and investigate the influence of the initial parameters of these states on the relative entropy of entanglement. In section III, we analyze the fidelity of teleportation for the pair cat states by using joint measurements of the photon-number sum and phase difference.

The influence of phase damping on the fidelity is discussed. A conclusion is given in section IV.

II. RELATIVE ENTROPY OF ENTANGLEMENT OF PAIR CAT STATES IN PHASE DAMPING CHANNEL

The relative entropy of entanglement is a good measure of quantum entanglement, it reduces to the Von Neumann entropy of the reduced density operator of either subsystems for pure states. For a mixed state ρ , the relative entropy of entanglement [15] is defined by $E_R(\rho) = \min_{\sigma \in D} S(\rho \parallel \sigma)$, where D is the set of all disentangled states, and $S(\rho \parallel \sigma) = \text{Tr}[\rho(\log_2 \rho - \log_2 \sigma)]$ is the quantum relative entropy. It is usually hard to calculate the relative entropy of entanglement for mixed states. Recently, it has been shown [27] that the relative entropy of entanglement for a class of mixed states characterized by the following density matrix

$$\rho = \sum_{n_1, n_2} a_{n_1, n_2} |\phi_{n_1}, \psi_{n_1}\rangle \langle \phi_{n_2}, \psi_{n_2}| \quad (1)$$

can be written as

$$E_R(\rho) = - \sum_n a_{n,n} \log_2 a_{n,n} + \text{Tr}(\rho \log_2 \rho). \quad (2)$$

The separate state ρ^* that minimizes the quantum relative entropy $S(\rho \parallel \rho^*)$ is

$$\rho^* = \sum_n a_{n,n} |\phi_n, \psi_n\rangle \langle \phi_n, \psi_n|, \quad (3)$$

where, $|\phi_n\rangle$ and $|\psi_n\rangle$ are orthogonal states of each subsystem. The states in Eq.(1) are also called maximally correlated states and are known to have some interesting properties. For example, the PPT distillable entanglement is exactly the same as the relative entropy of entanglement [9].

Now, we consider the phase damping model. The density matrix satisfies the following master equation in the interaction picture

$$\frac{d}{dt} \rho(t) = (L_1 + L_2) \rho(t), \quad (4)$$

with

$$L_i \rho = \frac{\gamma_i}{2} [2a_i^\dagger a_i \rho a_i^\dagger a_i - (a_i^\dagger a_i)^2 \rho - \rho (a_i^\dagger a_i)^2], \quad (5)$$

where, $\gamma_i (i = 1, 2)$ is the i th mode phase damping coefficient, and a_i^\dagger (a_i) is the creation (annihilation) operator of the i th mode field. For arbitrary initial states described by the density matrix ρ_0 , the solution of Eq.(4) can be obtained,

$$\rho(t) = \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_1! k_2!} (\gamma_1 t)^{k_1} (\gamma_2 t)^{k_2} \Lambda_{k_1, k_2}(t) \rho_0 \Lambda_{k_1, k_2}(t), \quad (6)$$

where

$$\Lambda_{k_1, k_2}(t) = (a_1^\dagger a_1)^{k_1} (a_2^\dagger a_2)^{k_2} \exp\left[-\frac{\gamma_1 t}{2} (a_1^\dagger a_1)^2 - \frac{\gamma_2 t}{2} (a_2^\dagger a_2)^2\right]. \quad (7)$$

If we assume the initial density matrix ρ_0 is arbitrary two-mode continuous variable pure states, i.e.,

$$\rho_0 = \sum_{n, m} \sum_{n', m'} a_{n, m} a_{n', m'}^* |n, m\rangle \langle n', m'|, \quad (8)$$

where $|n, m\rangle$ is two-mode particle number state. Then, the time-evolution density matrix with the initial condition is calculated as

$$\rho(t) = \sum_{n, m} \sum_{n', m'} a_{n, m} a_{n', m'}^* \exp\left[-\frac{\gamma_1 t}{2} (n - n')^2 - \frac{\gamma_2 t}{2} (m - m')^2\right] |n, m\rangle \langle n', m'|. \quad (9)$$

If the density matrix $\rho(t)$ in Eq.(9) can be expressed as the similar form of Eq.(1), i.e.,

$$\rho(t) = \sum_{n_1, n_2} c_{n_1, n_2}(t) |\phi'_{n_1}, \psi'_{n_1}\rangle \langle \phi'_{n_2}, \psi'_{n_2}|, \quad (10)$$

where, $|\phi'_n\rangle$ and $|\psi'_n\rangle$ are orthogonal states of modes 1 and 2, the relative entropy of entanglement of $\rho(t)$ in Eq.(10) can be expressed as,

$$E_R(\rho(t)) = -\sum_n c_{n, n}(t) \log_2 c_{n, n}(t) + \text{Tr}(\rho(t) \log_2 \rho(t)), \quad (11)$$

and the separate state ρ^* that minimizes the quantum relative entropy is

$$\rho^*(t) = \sum_n c_{n, n}(t) |\phi'_n, \psi'_n\rangle \langle \phi'_n, \psi'_n|. \quad (12)$$

In what follows, we investigate the relative entropy of entanglement of pair cat states in phase damping channel. Firstly, we will briefly outline the definition of pair cat states and the closely related pair coherent states. For two independent boson annihilation operators \hat{a}_1, \hat{a}_2 , a pair coherent state $|\xi, q\rangle$ is defined as an eigenstate of both the pair annihilation operator $\hat{a}_1 \hat{a}_2$ and the number difference operator $\hat{Q} = \hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2$ [16], i.e.,

$$\hat{a}_1 \hat{a}_2 |\xi, q\rangle = \xi |\xi, q\rangle, \quad \hat{Q} |\xi, q\rangle = q |\xi, q\rangle, \quad (13)$$

where ξ is a complex number and q is a fixed integer. Without loss of generality, we may set $q \geq 0$ and the pair coherent states can be explicitly expanded as a superposition of the two-mode Fock states, i.e.,

$$|\xi, q\rangle = N_q \sum_{n=0}^{\infty} \frac{\xi^n}{\sqrt{n!(n+q)!}} |n+q, n\rangle, \quad (14)$$

where $N_q = [|\xi|^{-q} I_q(2|\xi|)]^{-1/2}$ is the normalization constant and I_q is the modified Bessel function of the first kind of order q . It has been suggested by Reid and Krippner that the non-degenerate parametric oscillator transiently generates pair coherent states, in the limit of very large parametric nonlinearity and high-Q cavities[17]. Recently, Munro *et al.* have shown that the pair coherent states can be used to improve the detection sensitivity of weak forces[18]. Pair cat states $|\xi, q, \phi\rangle$ are proposed by Gerry

and Grobe [10], which are defined as superposition of two different pair coherent states, i.e.,

$$|\xi, q, \phi\rangle = N_\phi[|\xi, q\rangle + e^{i\phi}|\xi, q\rangle], \quad (15)$$

where the normalization constant N_ϕ is given by

$$N_\phi = \frac{1}{\sqrt{2}}[1 + N_q^2 \cos \phi \sum_{n=0}^{\infty} \frac{(-1)^n |\xi|^{2n}}{n!(n+q)!}]^{-\frac{1}{2}}. \quad (16)$$

It is easy to verify that the states $|\xi, q, \phi\rangle$ are eigenstates of the operator $(\hat{a}_1 \hat{a}_2)^2$ with eigenvalue ξ^2 . Gou et al. have proposed a scheme for generating the pair cat state of motion in a two-dimensional ion trap [12]. In their scheme, the trapped ion is excited bichromatically by five laser beams along different directions in the X-Y plane of the ion trap. Four of these have the same frequency and can be derived from the same source, reducing the demands on the experimentalist. It is shown that if the initial vibrational state is given by a two-mode Fock state, pair cat states are realized when the system reaches its steady state. Our following calculation show that pair cat states hold controllable entanglement. So, it is reasonable to regard the controlled two-dimensional trapped ion as a reliable source of entanglement. Recent achievements concerning the transfer of entangled state have provided us a possible way to map the pair cat state of the motional freedom of two dimensional trapped ions into freely propagating optical fields [13]. When the free photon propagates in the optical fibre, one of the encountered decoherence mechanisms is the phase damping. In the following, we discuss the entanglement of pair cat states in phase damping channel. We assume that the initial state is prepared in pair cat states $|\xi, q, \phi\rangle$. By making use of Eqs.(6) and (7), we obtain

$$\rho(t) = N_\phi^2 N_q^2 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{\exp[-\frac{\gamma_1 + \gamma_2}{2} t (n - m)^2] \xi^n \xi^{*m} (1 + (-1)^n e^{i\phi}) (1 + (-1)^m e^{-i\phi})}{\sqrt{n! m! (n+q)! (m+q)!}} |n+q, n\rangle \langle m+q, m|. \quad (17)$$

The relative entropy of entanglement for $\rho(t)$ is calculated as

$$E(t, \xi) = - \sum_n N_\phi^2 N_q^2 \frac{1 + (-1)^n e^{i\phi} |\xi|^2}{n! (n+q)!} \log_2 N_\phi^2 N_q^2 \frac{1 + (-1)^n e^{i\phi} |\xi|^2}{n! (n+q)!} \\ + \text{Tr}(\rho(t) \log_2 \rho(t)). \quad (18)$$

In numerical computations throughout this paper, the parameters $\gamma_1 = \gamma_2 = \gamma$, $d = \gamma t$ are chosen and the truncated photon number has been taken to be $\max(n) = \max(m) = 100$, the value of which is sufficiently large for numerical convergence. Figures 1,2 and 3 show that the relative entropy of entanglement E of the pair cat state increases with $|\xi|$ and decreases with degree of damping d , and can be controlled by adjusting the relative phase ϕ . This results can be explained as follows: the entanglement of pair cat states heavily depend on the photon number distribution which can be modified by the relative phase via the interference. Similar results have been obtained in Ref.[19]. In Fig.4, we plot the relative entropy of entanglement E of the pair cat state as a function of the relative phase ϕ for three values of the parameter q . Recently, Hiroshima has numerically calculated the relative entropy of entanglement

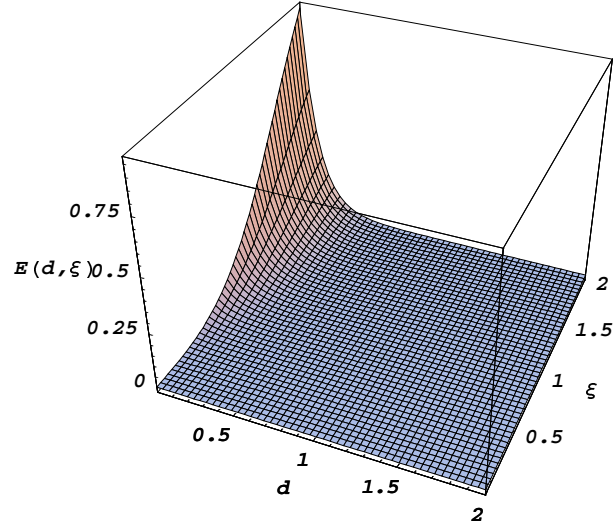


Figure 1: The relative entropy of entanglement E of the pair cat state as a function of the parameter $|\xi|$ and the degree of damping d for $q = 0$ with $\phi = \pi$.

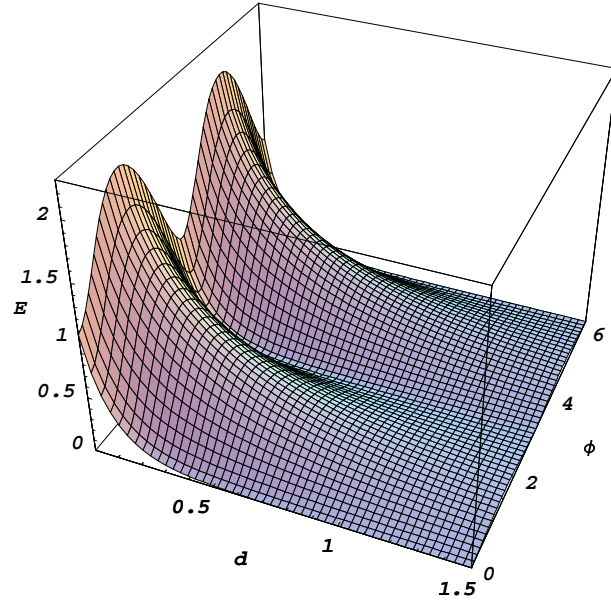


Figure 2: The relative entropy of entanglement E of the pair cat state as a function of the degree of damping d and the parameter ϕ for $q = 0$ with $|\xi| = 2$.

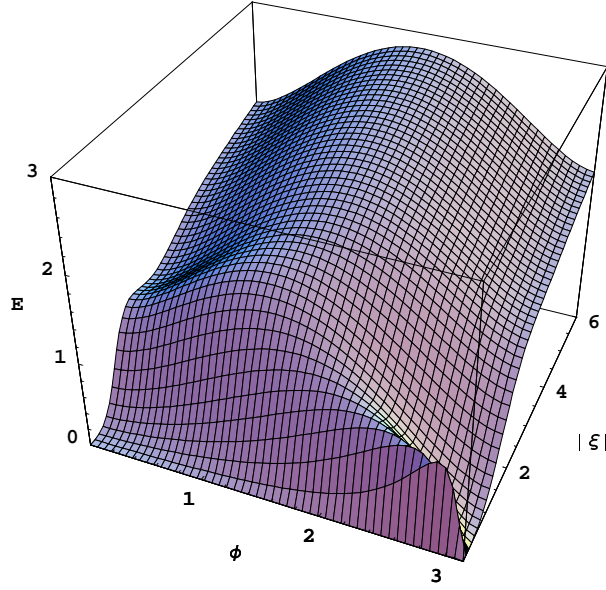


Figure 3: The relative entropy of entanglement E of the pair cat state as a function of the parameter $|\xi|$ and the parameter ϕ for $q = 0$ with $d = 0$.

of two-mode squeezed vacuum states, defined by $|\psi(r)\rangle = \exp[-r(a_1^\dagger a_2^\dagger - a_1 a_2)]|vac\rangle$, in phase damping channel [20]. It has been shown [21] that the two-mode squeezed vacuum state in phase damping channel is always distillable (and inseparable). In the following, we show that the pair cat states are always distillable (and inseparable) in phase damping channel.

For two-mode continuous variable states $|\psi\rangle$,

$$|\psi\rangle = \sum_n f_n |\phi_n, \psi_n\rangle, \quad (19)$$

where $|\phi_n\rangle$ and $|\psi_n\rangle$ are orthogonal particle number states of each subsystem and f_n satisfy the normalization condition $\sum_n |f_n|^2 = 1$. The density matrix with the initial condition $\rho(0) = |\psi\rangle\langle\psi|$ can be written as

$$\rho(t) = \sum_{n,m} f_n f_m^* \exp[-\frac{\gamma_1}{2}t(\phi_n - \phi_m)^2 - \frac{\gamma_2}{2}t(\psi_n - \psi_m)^2] |\phi_n, \psi_n\rangle\langle\phi_m, \psi_m|. \quad (20)$$

According to Ref.[22], if operator $\Omega(t) = \text{Tr}_D \rho(t) \otimes I - \rho(t)$ is not positive definite, there is always a scheme to distill $\rho(t)$. Here, we find $\text{Tr}_D \rho(t) = \sum_n |f_n|^2 |\phi_n\rangle\langle\phi_n|$. If there are two nonzero f_i, f_j , it is always possible to choose four vectors $|W_1\rangle = \frac{1}{\sqrt{2}}(|\phi_i, \psi_i\rangle + |\phi_j, \psi_j\rangle)$, $|W_2\rangle = \frac{1}{\sqrt{2}}(|\phi_i, \psi_i\rangle - |\phi_j, \psi_j\rangle)$, $|W_3\rangle = \frac{1}{\sqrt{2}}(|\phi_i, \psi_i\rangle + i|\phi_j, \psi_j\rangle)$, $|W_4\rangle = \frac{1}{\sqrt{2}}(|\phi_i, \psi_i\rangle - i|\phi_j, \psi_j\rangle)$. Then, we have

$$\Omega_1(t) \equiv \langle W_1 | \Omega(t) | W_1 \rangle = -\exp[-\frac{\gamma_1}{2}t(\phi_i - \phi_j)^2 - \frac{\gamma_2}{2}t(\psi_i - \psi_j)^2] \text{Re}(f_i f_j^*),$$

$$\Omega_2(t) \equiv \langle W_2 | \Omega(t) | W_2 \rangle = \exp[-\frac{\gamma_1}{2}t(\phi_i - \phi_j)^2 - \frac{\gamma_2}{2}t(\psi_i - \psi_j)^2] \text{Re}(f_i f_j^*),$$

$$\Omega_3(t) \equiv \langle W_3 | \Omega(t) | W_3 \rangle = \exp[-\frac{\gamma_1}{2}t(\phi_i - \phi_j)^2 - \frac{\gamma_2}{2}t(\psi_i - \psi_j)^2] \text{Im}(f_i f_j^*),$$

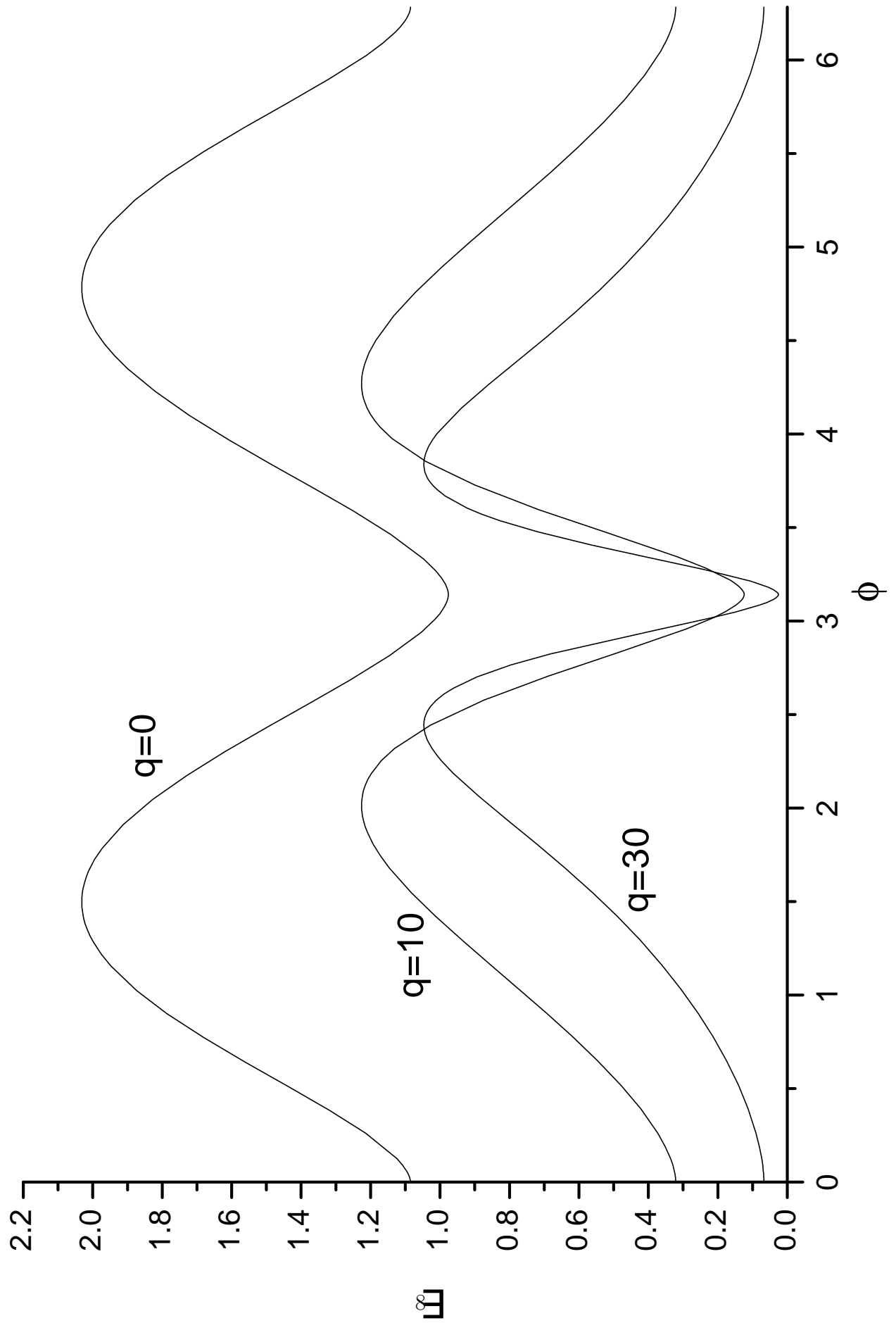


Figure 4: The relative entropy of entanglement E of the pair cat state as a function of the parameter ϕ for three values of $q = 0, 10$ and 30 with $d = 0$ and $|\xi| = 2$.

$$\Omega_4(t) \equiv \langle W_4 | \Omega(t) | W_4 \rangle = -\exp\left[-\frac{\gamma_1}{2}t(\phi_i - \phi_j)^2 - \frac{\gamma_2}{2}t(\psi_i - \psi_j)^2\right] \text{Im}(f_i f_j^*), \quad (21)$$

which satisfy

$$\begin{aligned} \Omega_1(t) + \Omega_2(t) &\equiv 0, \quad \Omega_3(t) + \Omega_4(t) \equiv 0, \\ \Omega_1(t) + i\Omega_4(t) &= -\exp\left[-\frac{\gamma_1}{2}t(\phi_i - \phi_j)^2 - \frac{\gamma_2}{2}t(\psi_i - \psi_j)^2\right] (f_i f_j^*) \neq 0, \\ \Omega_2(t) + i\Omega_3(t) &= \exp\left[-\frac{\gamma_1}{2}t(\phi_i - \phi_j)^2 - \frac{\gamma_2}{2}t(\psi_i - \psi_j)^2\right] (f_i f_j^*) \neq 0, \end{aligned} \quad (22)$$

Eqs.(22) show that there is at least one of $\Omega_k(t)$ ($k = 1, 2, 3, 4$), which is negative for any finite $\frac{\gamma_1 + \gamma_2}{2}t$. From the above, we obtain the following conclusion: the two-mode continuous variable state $|\psi\rangle = \sum_n f_n |\phi_n, \psi_n\rangle$, in which $|\phi_n\rangle$ and $|\psi_n\rangle$ are orthogonal particle number states of each subsystem, is always distillable (and inseparable) in phase damping channel, if there are at least two nonzero values of coefficients f_n . Obviously, pair coherent states and pair cat states which belong to the family of states in Eq.(20) is always distillable in phase damping channel. It should be interesting to consider a slightly modified purification protocol similar to the protocol in Ref.[14] to distill maximal entangled states from the mixed pair cat states or mixed pair coherent states due to phase damping.

III. FIDELITY OF TELEPORTATION VIA PAIR CAT STATES IN PHASE DAMPING CHANNEL

Recently, Cochrane et al. have presented a teleportation protocol by making use of joint measurements of the photon number sum and phase difference on two field modes [11]. Various kinds of two modes entangled states used as the entanglement resource have been discussed and the respective teleportation fidelities have been investigated. In this section, we adopt the protocol of cochrane et al. to investigate the fidelity of teleportation, in which the pair cat state is utilized as the entanglement resource. The influence of phase damping on the fidelity is also discussed.

Consider arbitrary target state sent by Alice to Bob

$$|\psi\rangle_T = \sum_{k=0}^{\infty} d_k |k\rangle_T, \quad (23)$$

where $|k\rangle_T$ is the fock state. Initially, Alice and Bob share the two-mode fields in the pair cat state. Then, the total state is

$$|\psi\rangle = N_\phi N_q \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{d_k \xi^n [1 + (-1)^n e^{i\phi}]}{\sqrt{n!(n+q)!}} |n+q\rangle_a |n\rangle_b |k\rangle_T. \quad (24)$$

The whole operation of this teleportation protocol can be decomposed as two steps: Alice makes a joint measurement of the photon number sum and phase difference of the target state and her component of the pair cat state; The results of the joint measurement are sent to Bob via the classical channel, and Bob reproduce the target

state after appropriate amplification and phase shift operations according to the results of the joint measurement. The joint measurement of the photon number sum and phase difference has attracted much attention due to its extensive potential applications both in quantum optics and quantum information [23,24]. In Ref.[24], Luis et al. introduced the hermitian phase-difference operator

$$\hat{\Theta}_{12} = \sum_{N=0}^{\infty} \sum_{r=0}^N \theta_r^N |\theta_r^N\rangle \langle \theta_r^N|, \quad (25)$$

with

$$|\theta_r^N\rangle = \frac{1}{\sqrt{N+1}} \sum_{n=0}^N e^{in\theta_r^N} |n\rangle_1 |N-n\rangle_2, \quad (26)$$

and

$$\theta_r^N = \vartheta + \frac{2\pi r}{N+1}, \quad (27)$$

where ϑ is an arbitrary angle. It is obvious that the joint measurement projects the two-mode quantum state onto $|\theta_r^N\rangle$. In Ref.[25], a physical scheme of the joint measurement of the photon number sum and phase difference of two-mode fields was proposed, in which only the linear optical elements and single-photon detector are involved.

If Alice measure the number sum $\hat{N}_a + \hat{N}_T$ of the target and her component of the pair cat state with result N , the state of the total system is projected onto

$$|\psi^{(N)}\rangle = \left[\frac{P(\xi, q, \phi, N)}{N_q^2 N_\phi^2} \right]^{-1/2} \sum_{l=q}^N \frac{d_{N-l} \xi^{l-q} [1 + (-1)^{l-q} e^{i\phi}]}{\sqrt{l!(l-q)!}} |l\rangle_a |l-q\rangle_b |N-l\rangle_T, \quad (28)$$

where

$$P(\xi, q, \phi, N) = N_q^2 N_\phi^2 \sum_{l=q}^N \frac{2|d_{N-l} \xi^{l-q}|^2 [1 + (-1)^{l-q} \cos \phi]}{l!(l-q)!}, \quad (29)$$

is the probability of obtaining the result N . Further measurement of phase difference with the result θ_- performed by Alice will project Bob's mode onto the pure state

$$|\psi^{(N, \theta_-)}\rangle = \left[\frac{P(\xi, q, \phi, N)}{N_q^2 N_\phi^2} \right]^{-1/2} \sum_{n=0}^{N-q} \frac{d_{N-q-n} (e^{-i\theta_-} \xi)^n [1 + (-1)^n e^{i\phi}]}{\sqrt{n!(n+q)!}} |n\rangle_b. \quad (30)$$

Alice sends the values N and θ_- to Bob, and then Bob amplifies his mode so that $|n\rangle_b \rightarrow |N-q-n\rangle_b$ [26] and makes a operation $e^{-i\hat{N}_b \theta_-}$ for phase shifting his mode. The teleportation protocol is then completed and Bob finally has the state in

$$|\psi^{(N)}\rangle_b = \left[\frac{P(\xi, q, \phi, N)}{N_q^2 N_\phi^2} \right]^{-1/2} \sum_{n=0}^{N-q} \frac{d_{N-q-n} \xi^n [1 + (-1)^n e^{i\phi}]}{\sqrt{n!(n+q)!}} |N-q-n\rangle_b. \quad (31)$$

The fidelity of this protocol depends on the result N and can be obtained as follows

$$F(\xi, q, \phi, N) = \left[\frac{P(\xi, q, \phi, N)}{N_q^2 N_\phi^2} \right]^{-1} \left| \sum_{n=0}^{N-q} \frac{|d_{N-q-n}|^2 \xi^n [1 + (-1)^n e^{i\phi}]}{\sqrt{n!(n+q)!}} \right|^2. \quad (32)$$

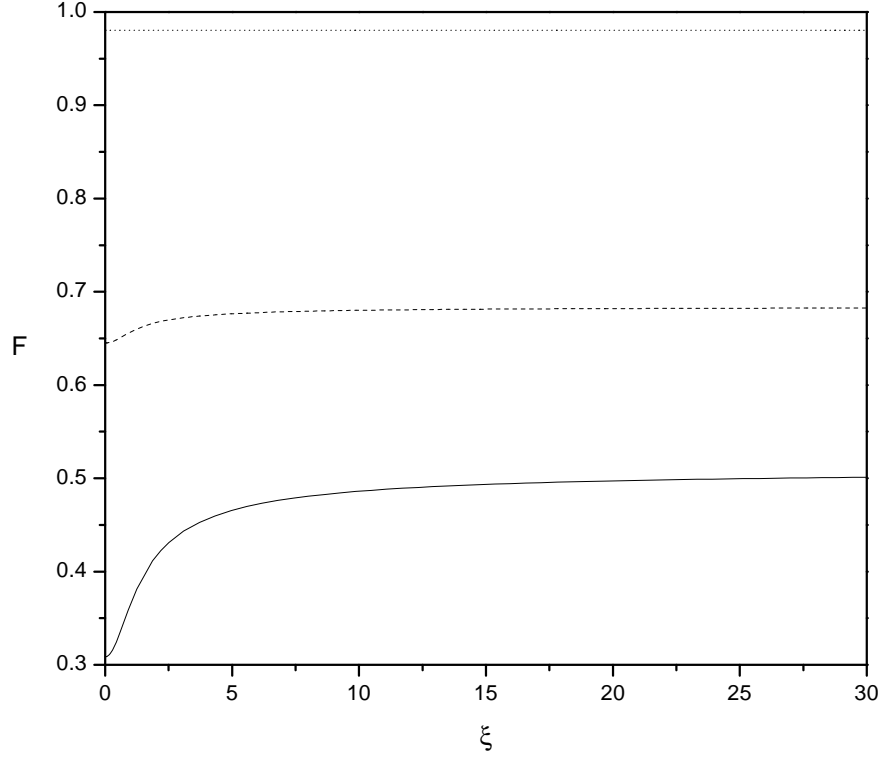


Figure 5: The average fidelity is plotted as the functions of the parameters ξ with $q = 0$ and $\phi = \pi/2$ for different values of $|\alpha|$, $|\alpha| = 1$ (Solid Line), $|\alpha| = 0.5$ (Dash Line), $|\alpha| = 0.1$ (Dot Line).

The average fidelity defined by $\bar{F}(\xi, q, \phi) \equiv \sum_{N=q}^{\infty} P(\xi, q, \phi, N) F(\xi, q, \phi, N)$ is

$$\bar{F}(\xi, q, \phi) = N_q^2 N_\phi^2 \sum_{N=0}^{\infty} \left| \sum_{n=0}^N \frac{|d_{N-n}|^2 \xi^n [1 + (-1)^n e^{i\phi}]}{\sqrt{n!(n+q)!}} \right|^2. \quad (33)$$

Let the target state be a coherent state $|\psi\rangle_T = |\alpha\rangle_T$. Then, the average fidelity can be expressed as

$$\bar{F}(\xi, q, \phi, \alpha) = N_q^2 N_\phi^2 e^{-2|\alpha|^2} \sum_{N=0}^{\infty} |\alpha|^{4N} \left| \sum_{n=0}^N \frac{|\alpha|^{-2n} \xi^n [1 + (-1)^n e^{i\phi}]}{(N-n)! \sqrt{n!(n+q)!}} \right|^2. \quad (34)$$

In Fig.5, we have plotted the average fidelity as the functions of the parameters ξ for different values of $|\alpha|$. It is shown that the average fidelity increases with the value of ξ . Furthermore, the average fidelity defined above heavily depends on the teleported states. If the teleported state is a coherent state, the smaller the amplitude of coherent states, the higher the average fidelity. Its physical reason can be elucidated by two facts: one fact is that this protocol works perfectly if the target is a number state [11]; the other fact is that the smaller the amplitude of a coherent state, the closer the state distance between the coherent state and a specific number state, i.e., the vacuum state. In what follows, we discuss the influence of phase damping on the fidelity of the above teleportation protocol. In this case, the state of the total system can be written as

follows

$$\rho_T = \sum_{k,l,n,m=0}^{\infty} \frac{d_k d_l^* \xi^n \xi^{*m} [1 + (-1)^n e^{i\phi}] [1 + (-1)^m e^{-i\phi}] e^{-\gamma t(n-m)^2}}{\sqrt{n!m!(n+q)!(m+q)!}} |k\rangle_{TT} \langle l| \otimes |n+q, n\rangle \langle m+q, m|. \quad (35)$$

After completing the protocol described above, Bob finally achieves the state in his mode expressed by

$$\rho_b = \left[\frac{P(\xi, q, \phi, N)}{N_q^2 N_\phi^2} \right]^{-1} \sum_{n,m=0}^{N'} \frac{d_{N'-n} d_{N'-m}^* \xi^n \xi^{*m} [1 + (-1)^n e^{i\phi}] [1 + (-1)^m e^{-i\phi}] e^{-\gamma t(n-m)^2}}{\sqrt{n!m!(n+q)!(m+q)!}} |N'-n\rangle_{bb} \langle N'-m|, \quad (36)$$

where $N' = N - q$ and N is the measured number sum of Alice's joint measurement. Then, the fidelity of this protocol depends on the result N and is

$$F(\xi, q, \phi, N, \gamma t) = \left[\frac{P(\xi, q, \phi, N)}{N_q^2 N_\phi^2} \right]^{-1} \sum_{n,m=0}^{N'} \frac{|d_{N'-n}|^2 |d_{N'-m}|^2 \xi^n \xi^{*m} [1 + (-1)^n e^{i\phi}] [1 + (-1)^m e^{-i\phi}] e^{-\gamma t(n-m)^2}}{\sqrt{n!m!(n+q)!(m+q)!}}. \quad (37)$$

The average fidelity is

$$\bar{F}(\xi, q, \phi, \gamma t) = N_q^2 N_\phi^2 \sum_{N'=0}^{\infty} \sum_{n,m=0}^{N'} \frac{|d_{N'-n}|^2 |d_{N'-m}|^2 \xi^n \xi^{*m} [1 + (-1)^n e^{i\phi}] [1 + (-1)^m e^{-i\phi}] e^{-\gamma t(n-m)^2}}{\sqrt{n!m!(n+q)!(m+q)!}}. \quad (38)$$

For a coherent state $|\alpha\rangle$ teleported by Alice, the average fidelity can be rewritten as

$$\bar{F}(\xi, q, \phi, |\alpha|, \gamma t) = N_q^2 N_\phi^2 e^{-2|\alpha|^2} \sum_{N'=0}^{\infty} |\alpha|^{4N'} \sum_{n,m=0}^{N'} \frac{|\alpha|^{-2n-2m} \xi^n \xi^{*m} [1 + (-1)^n e^{i\phi}] [1 + (-1)^m e^{-i\phi}] e^{-\gamma t(n-m)^2}}{(N'-n)!(N'-m)! \sqrt{n!m!(n+q)!(m+q)!}}. \quad (39)$$

In Fig.6, the average fidelity for the coherent states is plotted as a function of γt for different values of amplitude $|\alpha|$. It is shown that the phase damping deteriorate the average fidelity of teleportation basing on the pair cat state, which is qualitatively consistent with the behavior of its relative entropy of entanglement in the phase damping channel. As mentioned above, the essential parts of this teleportation protocol are the preparation of the pair cat state and the joint measurement of the number sum and phase difference. The direct preparation of pair cat states in two modes optical fields is still a open question. However, we can map the pair cat state of the motional freedom of two dimensional trapped ions into freely propagating optical fields. The details will be discussed elsewhere. Recently, the physical implementation of joint measurement of photon number sum and phase difference of two-mode optical fields is shown to be possible by using only the linear optical elements and the single-photon detector [25]. So we can conclude that the physical realization of this teleportation protocol is feasible at the present technology.

IV. CONCLUSION

In this paper, we investigate the entanglement of pair cat states in the phase damping channel by employing the relative entropy of entanglement. We give the

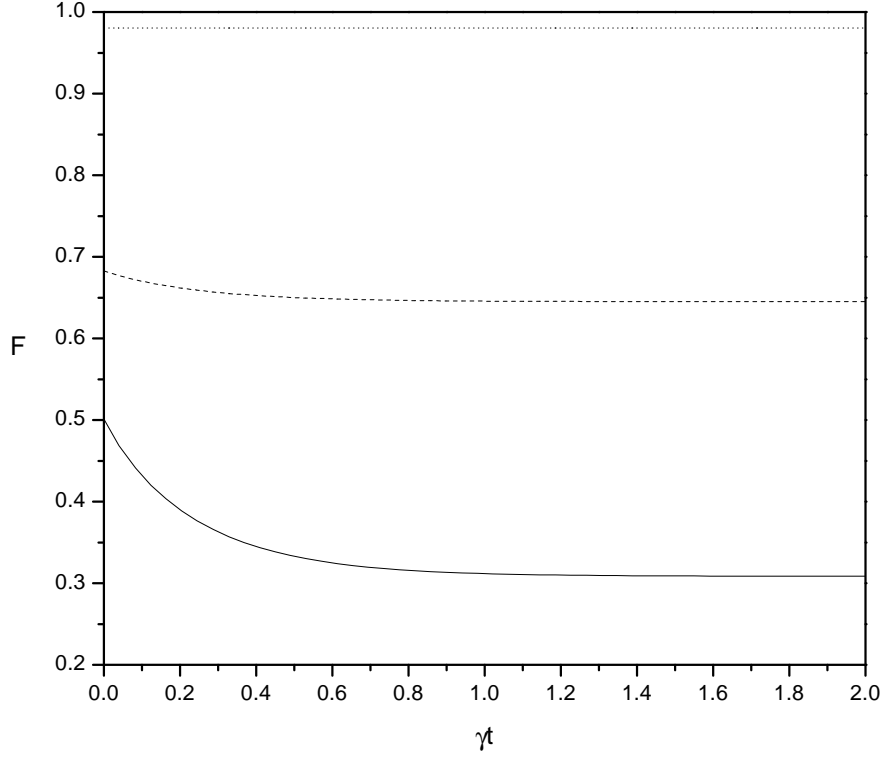


Figure 6: The average fidelity is plotted as the functions of the phase damping coefficient γt with $\xi=30$, $q = 0$ and $\phi = \pi/2$ for different values of $|\alpha|$, $|\alpha| = 1$ (Solid Line), $|\alpha| = 0.5$ (Dash Line), $|\alpha| = 0.1$ (Dot Line).

numerical calculations of the relative entropy of entanglement of this state in the phase damping channel and study the influence of the parameters on the relative entropy of entanglement. We find that the relative phase of the pair cat state can control the relative entropy of entanglement. Then, we show that the pair cat states can always be distillable in the phase damping channel. Finally, we analyze the fidelity of teleportation for the pair cat states by using joint measurements of the photon-number sum and phase difference. The influence of phase damping on the fidelity is discussed. The behavior of average fidelity for teleporting a coherent based on the pair cat state is qualitatively consistent with its relative entropy of entanglement. It is interesting to investigate the entanglement and teleportation fidelity of pair cat states in the amplitude damping channel.

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